### Expressing Liveness

Liveness properties are best described using the **AF** or **F** combinators:

- **AF** done
- **F** done
- **AG** (req ⇒ **AF** grant)
- **G** (req ⇒ **F** grant)
- **AG AF** tick
- **G F** tick

### Liveness Properties

A **liveness property** is a property stating that

“something good will eventually happen.”

### Combining Liveness Properties

**Some Rules:**

- \( F \varphi \lor F \psi \equiv F (\varphi \lor \psi) \)
- \( AF \varphi \lor AF \psi \equiv AF (\varphi \lor \psi) \)

**But:**

- \( F \varphi \land F \psi \not\equiv F (\varphi \land \psi) \)
- \( AF \varphi \land AF \psi \not\equiv AF (\varphi \land \psi) \)

### Examples

- “The light will turn green.”
- “If the key is in the ignition position, the car will start eventually.”
- “The program will terminate.”
- “The clock will tick infinitely often.”

**Repeated Liveness Fairness**

### What about Until?

\( \varphi \mathcal{U} \psi \equiv (\varphi \mathcal{W} \psi) \land F \psi \)

Safety  
Liveness

Linear temporal logic formulas can be written as a conjunction of a safety and a liveness formula.
Nonconflicting

Recall:
A system is nonconflicting if, from every reachable state, a marked state can be reached.

CTL Formula:
\[
\text{AG EF marked\_state}
\]

Is this a liveness property?

The Dual Property

“The system will become stable.”

\[
\text{FG stable}
\]

\[
\text{stable} \rightarrow \text{stable} \rightarrow \text{stable} \rightarrow \text{stable} \rightarrow \ldots
\]

Bounded Liveness

A bounded liveness property is a liveness property that comes with a maximal delay within which the “good thing” must occur.

Example:
“There will be rain within three days.”

Such properties are safety properties!

So why study liveness?

Example of Repeated Liveness

“If access to the critical section is infinitely often requested, then access will be granted infinitely often.”

\[
\text{A (GF req } \Rightarrow \text{GF grant)}
\]

\[
\text{A (FG \neg\text{req } \lor \text{GF grant)}
\]

Repeated Liveness

“The clock ticks infinitely often.”

\[
\text{GF tick}
\]

\[
\text{AG AF tick}
\]

\[
\text{tick} \rightarrow \text{tick} \rightarrow \text{tick} \rightarrow \text{tick} \rightarrow \ldots
\]

More Examples

- “If the train is on track section A, then it will eventually reach track section C.”
- “If the tank is empty, the outlet valve will eventually be closed.”
- “If a user requests access to the critical section, they will eventually be granted access.”