

COMP 140-04B

Foundations of Computer Science

Model Answers for Test 3

30th September 2004

Family Name: _____

Given Name: _____

ID Number: _____

Instructions

1. First write your name and ID number into the space above.
2. Write your answers into the space provided.
3. There are five questions, worth 10 marks each. Answer all of them.
4. Time allowed is 40 minutes.
5. Do *not* fill the table below.

1	2	3	4	5	Total

Question 1 (5+5 marks)

For each of the following well-formed formulas of propositional logic, determine by using truth tables whether it is valid, satisfiable but not valid, or unsatisfiable.

a) $(p \rightarrow (q \vee r)) \rightarrow ((p \wedge \neg r) \rightarrow q)$

b) $(p \rightarrow (q \wedge r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

Answer:

a)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$\neg r$	$p \wedge \neg r$	$(p \wedge \neg r) \rightarrow q$	" \rightarrow "
F	F	F	F	T	T	F	T	T
F	F	T	T	T	F	F	T	T
F	T	F	T	T	T	F	T	T
F	T	T	T	T	F	F	T	T
T	F	F	F	F	T	T	F	T
T	F	T	T	T	F	F	T	T
T	T	F	T	T	T	T	T	T
T	T	T	T	T	F	F	T	T

(4 marks)

The formula is valid.

(1 mark)

b)

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	" \leftrightarrow "
F	F	F	F	T	F	T	T
F	F	T	F	T	F	T	T
F	T	F	F	T	F	T	T
F	T	T	T	T	F	T	T
T	F	F	F	F	F	T	F
T	F	T	F	F	F	T	F
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

(4 marks)

The formula is satisfiable but not valid.

(1 mark)

Question 2 (2+4+4 marks)

Find minimal disjunctive normal forms for the propositional formulas represented by each of the following Karnaugh Maps.

a)

	p		
	T	T	
r		T	
		q	

b)

	p			
	T			
r	T	T	T	T
	T	T	T	
				s
		q		

c)

	p			
	T			T
r		T		
			T	
	T			T
				s
		q		

Answer:

$$a) (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge q)$$

(2 marks)

$$b) (\neg p \wedge \neg q \wedge \neg s) \vee (\neg q \wedge r) \vee (p \wedge r) \vee (r \wedge \neg s)$$

(4 marks)

$$c) (\neg p \wedge \neg r) \vee (p \wedge \neg q \wedge r \wedge \neg s) \vee (p \wedge q \wedge r \wedge s)$$

(4 marks)

Question 3 (3+7 marks)

Use the laws of equivalence to obtain a minimal disjunctive normal form for each of the following well-formed formulas. Write down each transformation step in a line of its own and indicate the law of equivalence which you have used.

a) $(p \wedge q) \rightarrow (r \wedge s)$

b) $p \leftrightarrow (p \rightarrow q)$

Answer:

a) $(p \wedge q) \rightarrow (r \wedge s)$
 $\neg(p \wedge q) \vee (r \wedge s)$ (*Definition of \rightarrow*)
 $\neg p \vee \neg q \vee (r \wedge s)$ (*De Morgan's Law*)

(3 marks)

b) $p \leftrightarrow (p \rightarrow q)$
 $(p \rightarrow (p \rightarrow q)) \wedge ((p \rightarrow q) \rightarrow p)$ (*Definition of \leftrightarrow*)
 $(\neg p \vee \neg p \vee q) \wedge (\neg(\neg p \vee q) \vee p)$ (*Definition of \rightarrow*)
 $(\neg p \vee q) \wedge (\neg(\neg p \vee q) \vee p)$ (*Idempotence*)
 $(\neg p \vee q) \wedge ((\neg\neg p \wedge \neg q) \vee p)$ (*De Morgan's Law*)
 $(\neg p \vee q) \wedge ((p \wedge \neg q) \vee p)$ (*Double Negation*)
 $(\neg p \vee q) \wedge p$ (*Absorption*)
 $(\neg p \wedge p) \vee (q \wedge p)$ (*Distributivity*)
 $false \vee (q \wedge p)$ (*Excluded Middle*)
 $q \wedge p$ (*Domination*)

(7 marks)

Question 4 (5+5 marks)

Formalise the following arguments in propositional logic, and state whether they are correct or incorrect.

- a) If Mr. or Mrs. Smith earns more than \$30,000 per year, then the Smith family can afford holidays in Hawaii.
 Either Mr. Smith or his wife (but not both of them) earns more than \$30,000 per year.
 Therefore, the Smith family can afford holidays in Hawaii.
- b) If the sky is clear and the stars are visible, then Max is at the observatory.
 Unless Max is not at the observatory, the sky is clear.
 Therefore, the sky is clear if and only if Max is at the observatory.

Answer:

a) *Let*

$$\begin{aligned} r &= \text{“Mr. Smith earns more than \$30,000 per year.”} \\ s &= \text{“Mrs. Smith earns more than \$30,000 per year.”} \\ h &= \text{“The Smith family can afford holidays in Hawaii.”} \end{aligned}$$

Then we can formalise the argument as follows.

$$\text{Premise 1: } (r \vee s) \rightarrow h \quad \text{(1 mark)}$$

$$\text{Premise 2: } r \oplus s \quad \text{(1 mark)}$$

$$\text{Conclusion: } h \quad \text{(1 mark)}$$

$$\text{This argument is correct.} \quad \text{(2 marks)}$$

b) *Let*

$$\begin{aligned} c &= \text{“The sky is clear.”} \\ v &= \text{“The stars are visible.”} \\ o &= \text{“Max is at the observatory.”} \end{aligned}$$

Then we can formalise the argument as follows.

$$\text{Premise 1: } (c \wedge v) \rightarrow o \quad \text{(1 mark)}$$

$$\text{Premise 2: } \neg\neg o \rightarrow c \quad \text{(1 mark)}$$

$$\text{Conclusion: } c \leftrightarrow o \quad \text{(1 mark)}$$

$$\text{This argument is not correct.} \quad \text{(2 marks)}$$

Question 5 (4+2+2+2 marks)

In this question, we consider formulas using \rightarrow as the only connective. Consider the deductive system **D** consisting of the following axiom and rule.

$$(\mathbf{Axiom}) \quad A \rightarrow A \quad (\mathbf{Rule}) \quad \frac{A}{B \rightarrow A}$$

- Give a proof for the formula $q \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow q))$ in **D**. Write each proof step on a numbered line of its own, and indicate how it was obtained.
- Is **D** sound? Explain your answer briefly.
- State a valid formula using \rightarrow as the only connective that is not provable in **D**.
- Is **D** complete? Explain your answer briefly.

Answer:

- $(p \rightarrow q) \rightarrow (p \rightarrow q)$ **Axiom**
 - $q \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow q))$ **Rule(1)**

(4 marks)

b) **D** is sound,

(1 mark)

because **(Axiom)** is a valid formula and the immediate consequence of **(Rule)** is a logical consequence of its premise.

(1 mark)

c) For example: $p \rightarrow (q \rightarrow p)$.

(2 marks)

d) **D** is not complete.

(1 mark)

As shown under c), there exists a valid formula that cannot be proven by this deductive system.

(1 mark)