Causal Process Algebra

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Shadowy formal methods

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- The better the formal method the stronger the light and the darker the shadows
- Looking in the shadows is no criticism

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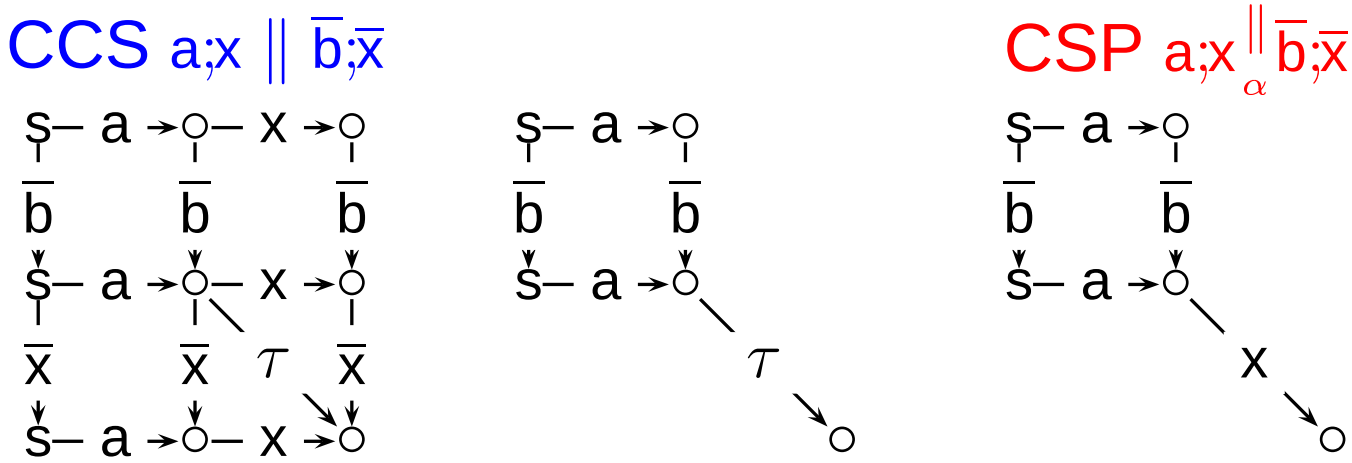
- process algebras communication abstracts away:
  1. one action causing another
  2. one process choosing

Process Model

- One set (kind) of observable actions
- communication or synchronisation:
  1. is exclusive - must occur if it can.
  2. is private - unobserved



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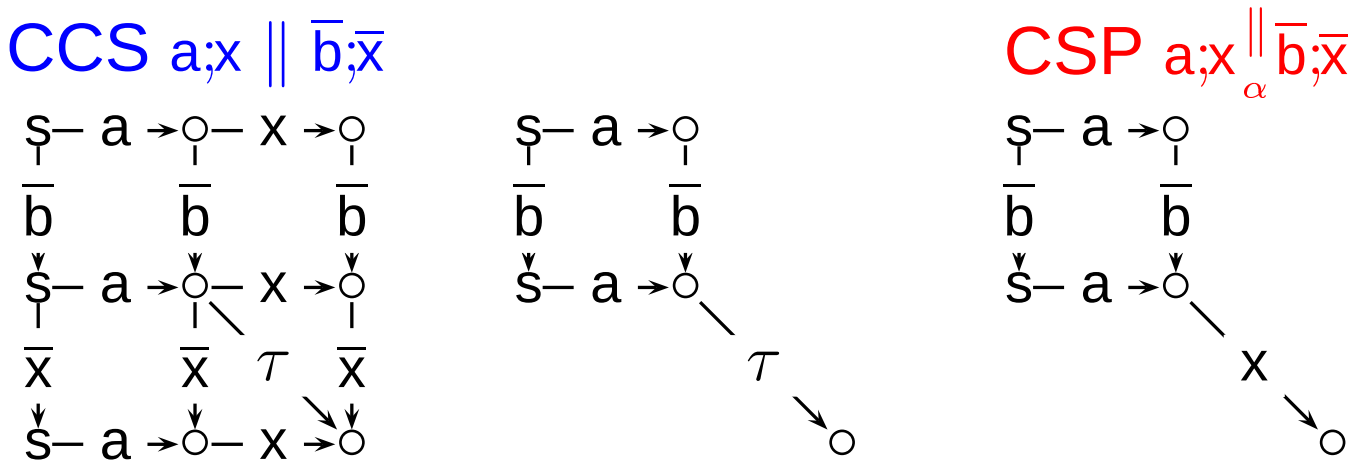

- CCS VM || Rob$\backslash\alpha$ called **restricted Composition** by Milner
- CSP $(\text{VM} \parallel_{\alpha} \text{Rob})/\alpha$

Causal Process Algebra

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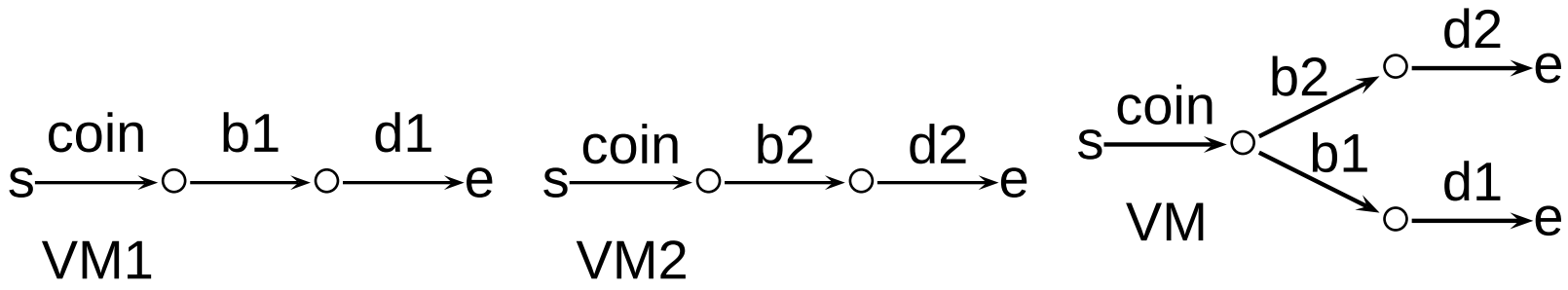
- Four advantages:
  1. extend refinement
  2. determinism by construction (Milner)
  3. small deterministic processes are implementable
  4. hiding private communication does not introduce nondeterminism

Example

- A world of Vending machines and Robots!



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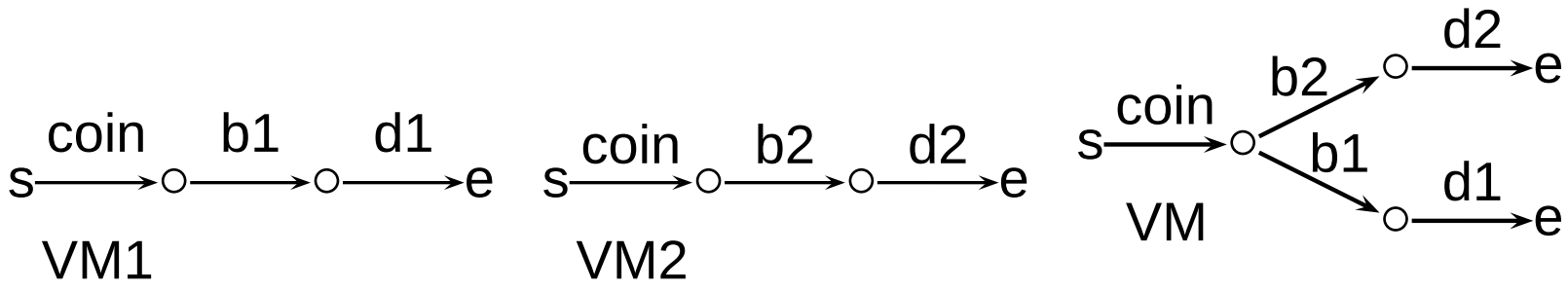

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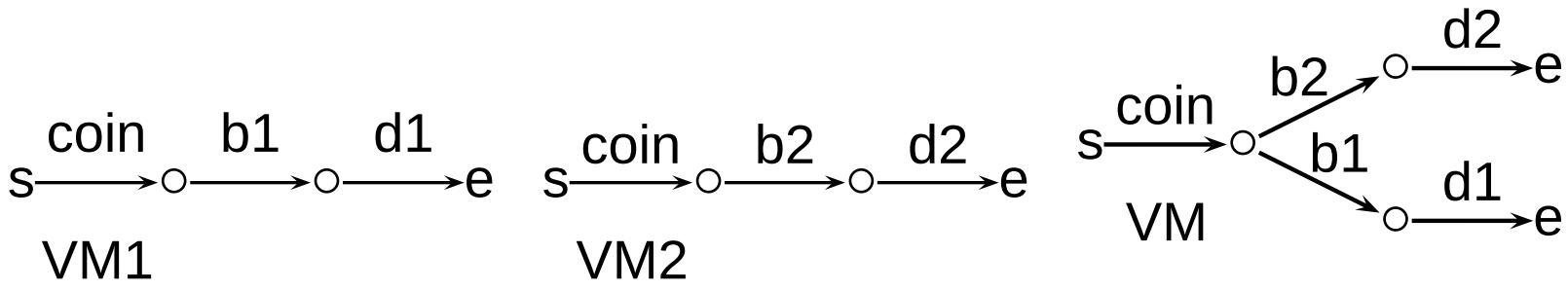

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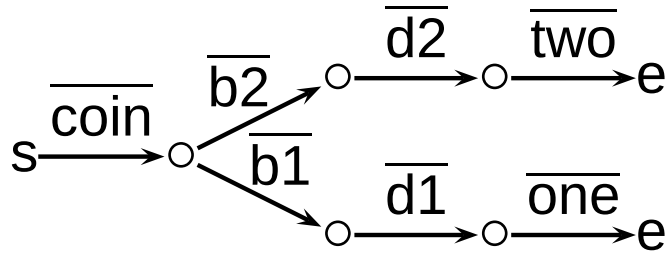
- Unsatisfiable in process algebra?
- Processes are at the wrong level of detail?

Solutions



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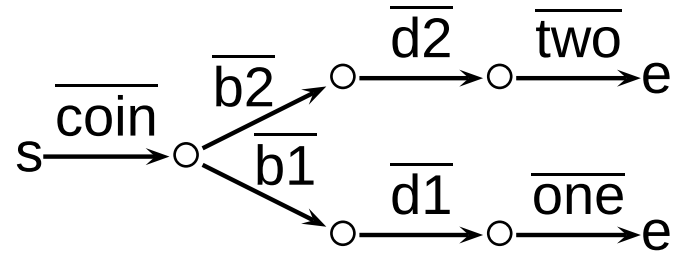

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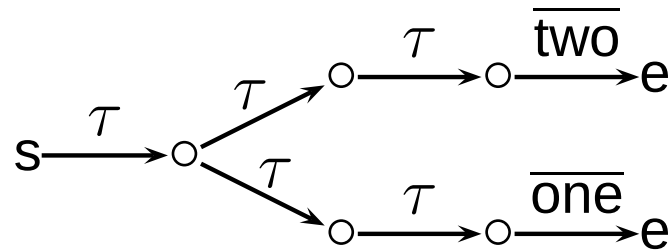

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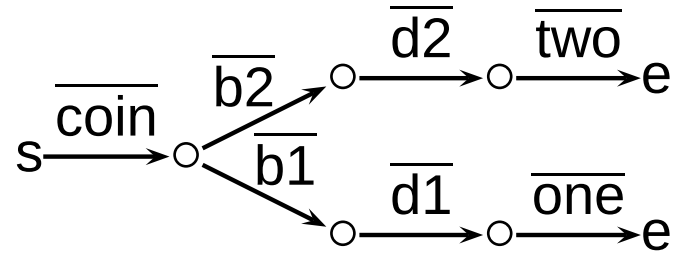

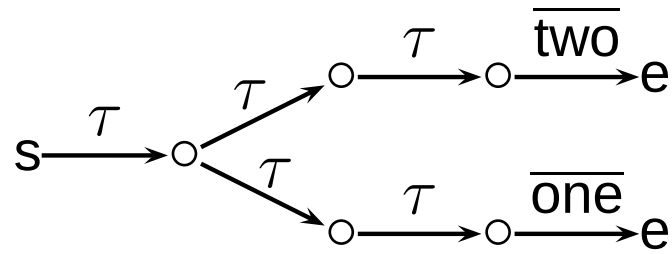

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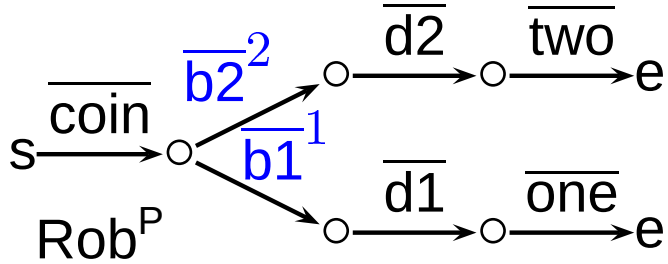

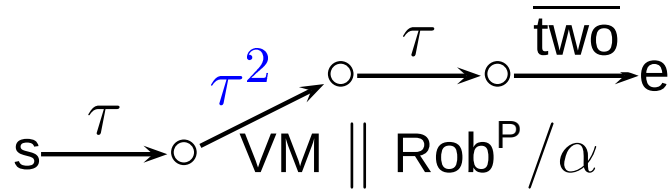

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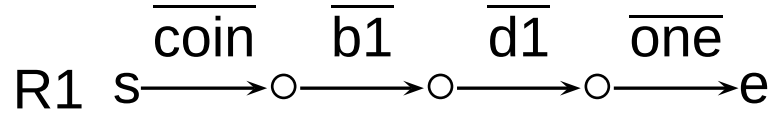

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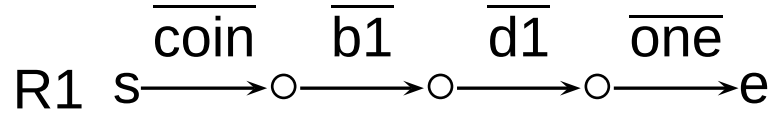

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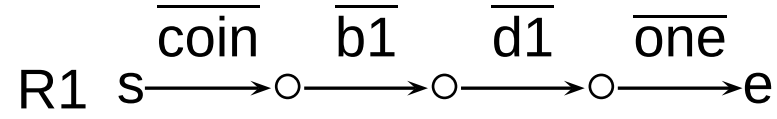

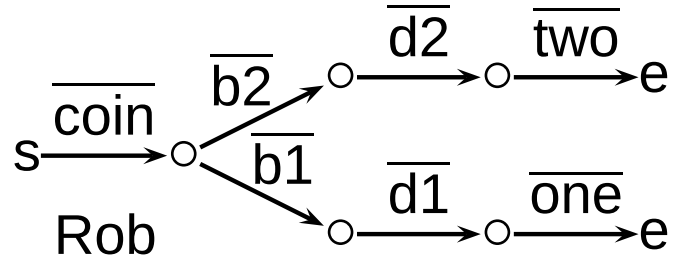

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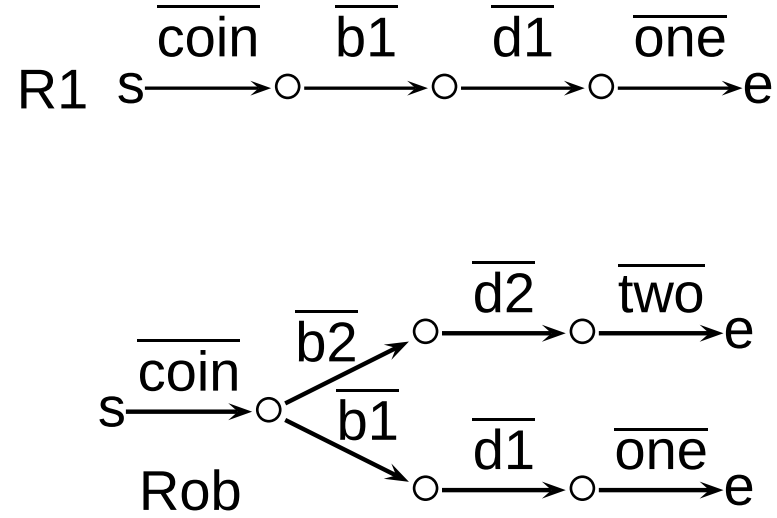

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  requires our process model be rebuilt.

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- Milner [**?**, p232] express the hope of finding “design rules, not too restrictive, which ensure that from determinate components we must arrive at a determinate system”
- defines *confluence* [**?**, p237] “to strengthen determinacy in such a way that it will be preserved by restricted Composition”

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- Hence CSP/CCS Rob and VM not implementable!
- Occam does not implement CSP/CCS style parallel composition
- Occam is consistent with CSP failure refinement:

$$(\mathsf{VM} \underset{\alpha}{\|} \mathsf{Rob})/\alpha \sqsubseteq_{F} \mathsf{VM} \|_{Occam} \mathsf{Rob}$$

- Not consistent with may and must testing refinement:

$$(\mathsf{VM} \underset{\alpha}{\|} \mathsf{Rob})/\alpha \not\sqsubseteq_{Test} \mathsf{VM} \|_{Occam} \mathsf{Rob}$$

CPA

- a set of names *Names*:
- **passive** actions $Act \stackrel{\text{def}}{=} \{\mathrm{a} \mid \mathrm{a} \in Names\}$ and
- **active** actions $\overline{Act} \stackrel{\text{def}}{=} \{\overline{\mathrm{a}} \mid \mathrm{a} \in Names\}$
- irreflexive priority relation $\angle \subseteq Pri \times Pri$
- priority function $Act_{Pri} : Names \longrightarrow Pri$
- function and relation lifted to actions or transitions.
- PLTS

$$T_{\mathrm{A}} \subseteq \{(n, (\mathrm{x}, p), m) \mid n, m \in N_{\mathrm{A}} \wedge (\mathrm{x}, p) \in Act_{Pri} \wedge \mathrm{x} \in Obs\} \cup \{(n, (\tau, p), m) \mid p \in Pri\}$$

$$n \xrightarrow{(\mathrm{x},p)} m$$

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$$x \xrightarrow{(\ddot{\mathrm{a}},p)} z \wedge x \xrightarrow{(\overline{\mathrm{b}},p)} y \Rightarrow z = y \wedge \ddot{\mathrm{a}} = \overline{\mathrm{b}}$$

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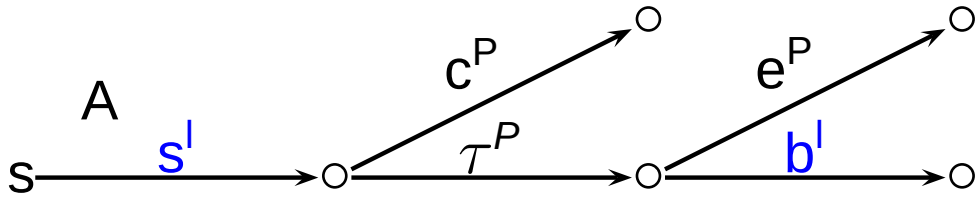
- We define $\preceq$: $Act_{Pri} \times Act_{Pri}$ by $(x, p) \preceq (y, p') \stackrel{\text{def}}{=} p \angle p'$.

  Define $\sqsubseteq_p \subseteq Act_{Pri} \times Act_{Pri}$ by $Act_{Pri1} \sqsubseteq_p Act_{Pri2} \stackrel{\text{def}}{=} \preceq_1 \subseteq \preceq_2$.
  Lift $\sqsubseteq_p$ to processes.
  $\sqsubseteq_{tp}$ is the transitive closure of $\sqsubseteq_p \cup \sqsubseteq_t$

Observational semantics



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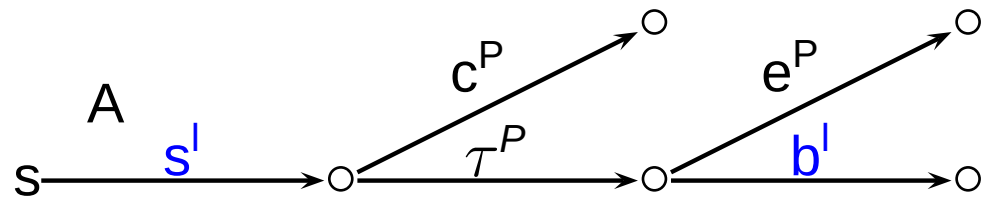

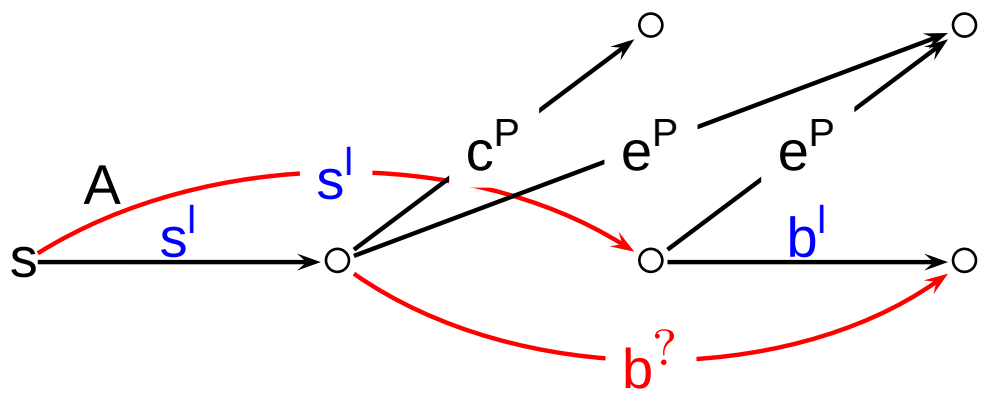

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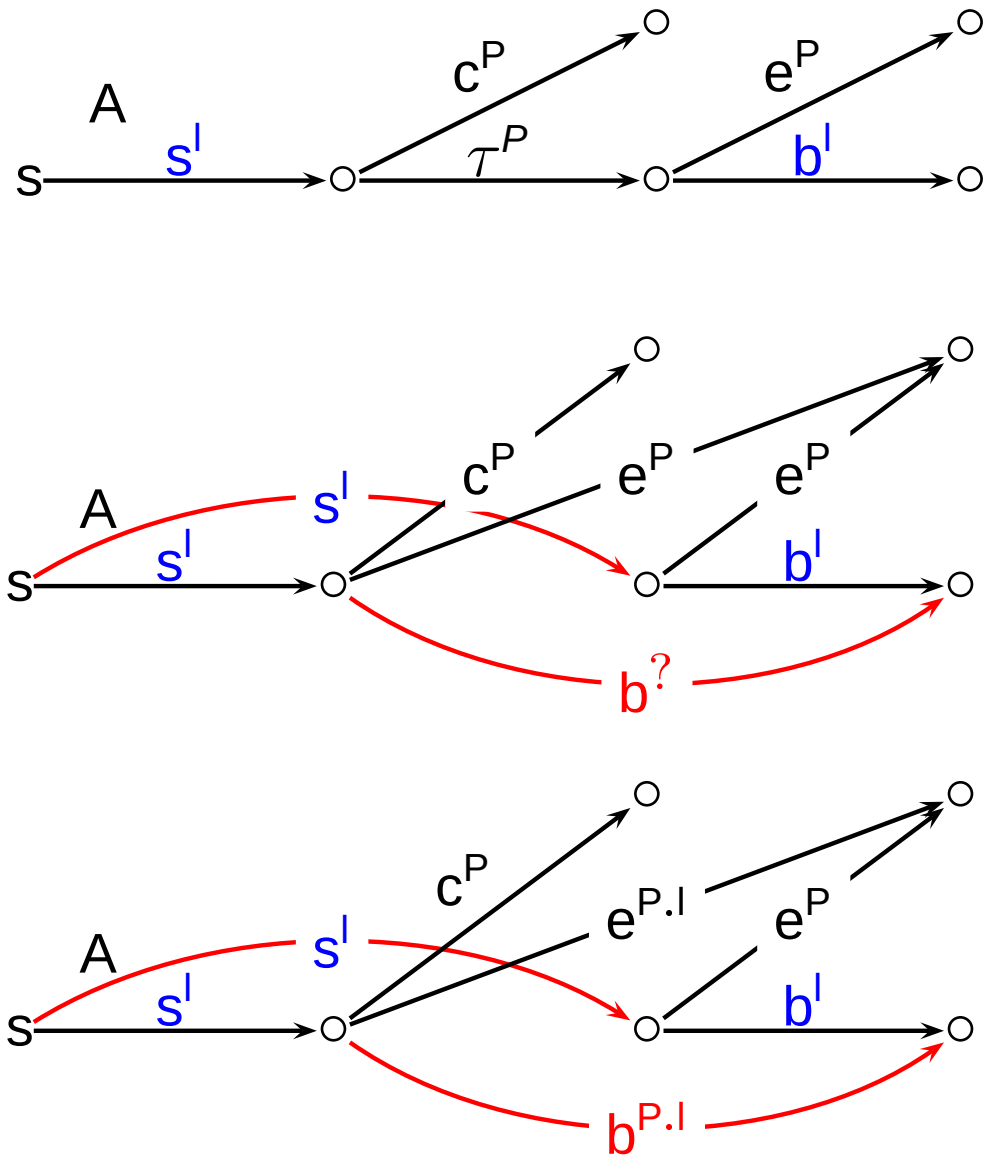
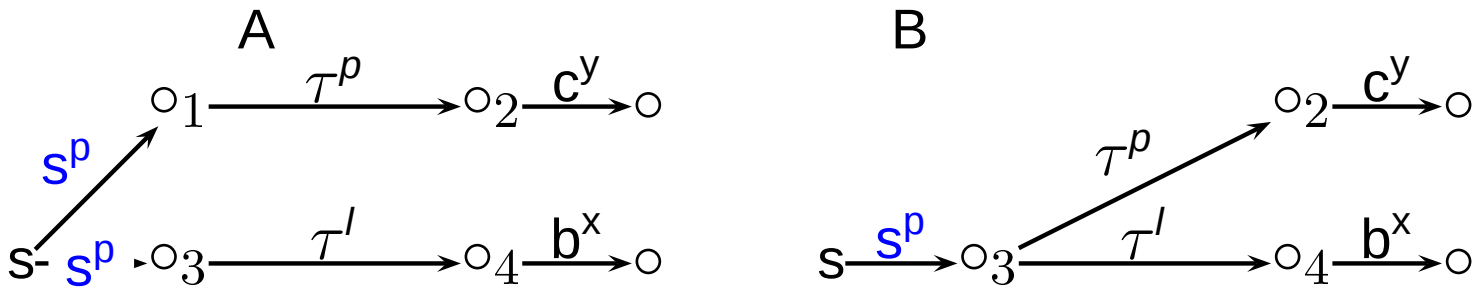


Observational semantics(2)



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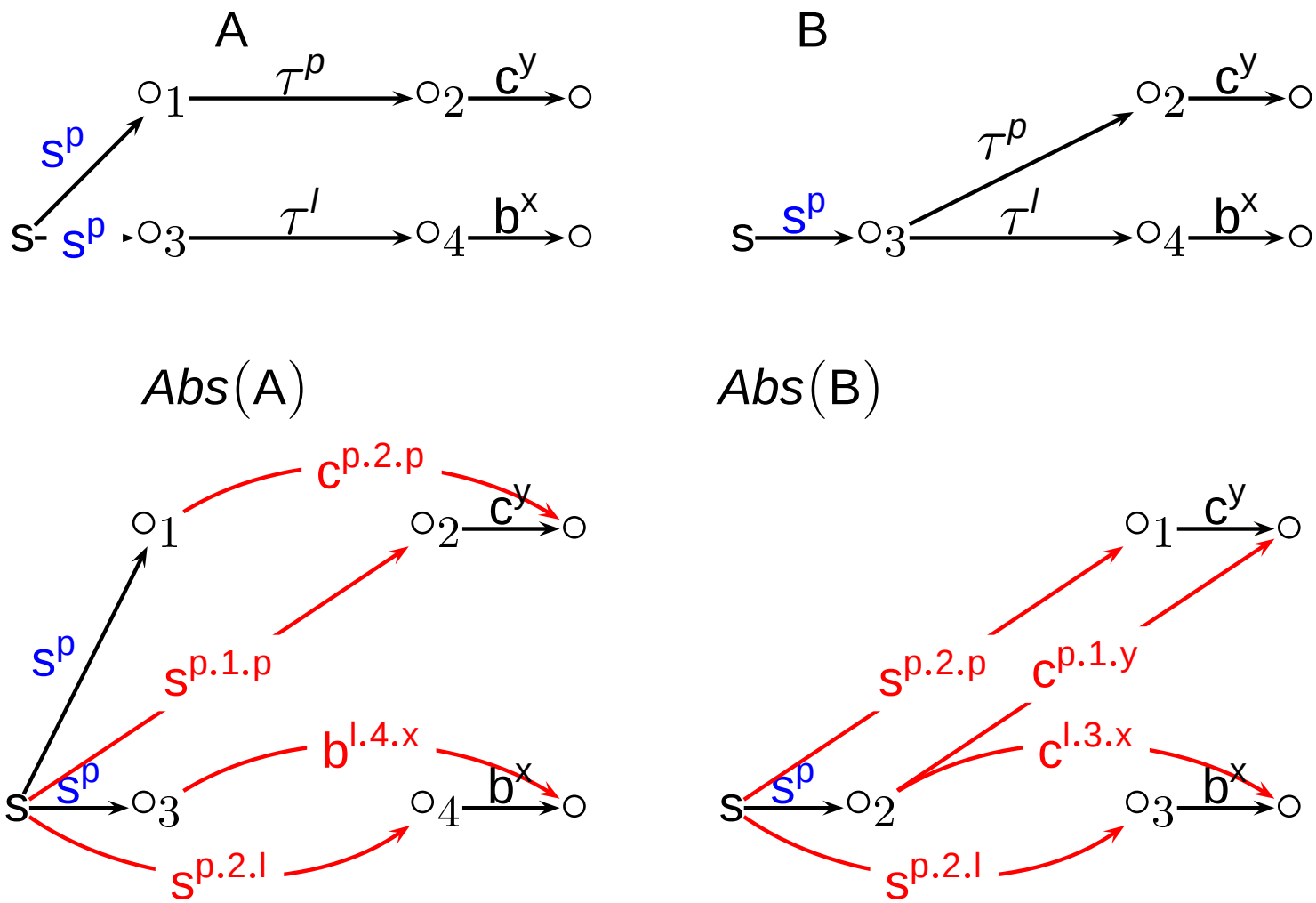

End

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- + priority
- Disadvantage more complexe semantics
- Four advantages:
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