Data Mining
Part 3

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Output: Knowledge representation

- Decision tables
- Decision trees
- Decision rules
- Association rules
- Rules with exceptions
- Rules involving relations
- Linear regression
- Trees for numeric prediction
- Instance-based representation
- Clusters
Output: representing structural patterns

Many different ways of representing patterns
  Decision trees, rules, instance-based, …
Also called “knowledge” representation
Representation determines inference method
Understanding the output is the key to understanding the underlying learning methods
Different types of output for different learning problems (e.g. classification, regression, …)
Decision tables

Simplest way of representing output:
Use the same format as input!

Decision table for the weather problem:

<table>
<thead>
<tr>
<th>Outlook</th>
<th>Humidity</th>
<th>Play</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td>High</td>
<td>No</td>
</tr>
<tr>
<td>Sunny</td>
<td>Normal</td>
<td>Yes</td>
</tr>
<tr>
<td>Overcast</td>
<td>High</td>
<td>Yes</td>
</tr>
<tr>
<td>Overcast</td>
<td>Normal</td>
<td>Yes</td>
</tr>
<tr>
<td>Rainy</td>
<td>High</td>
<td>No</td>
</tr>
<tr>
<td>Rainy</td>
<td>Normal</td>
<td>No</td>
</tr>
</tbody>
</table>

Main problem: selecting the right attributes
Decision trees

“Divide-and-conquer” approach produces tree
Nodes involve testing a particular attribute
Usually, attribute value is compared to constant
Other possibilities:
  - Comparing values of two attributes
  - Using a function of one or more attributes
Leaves assign classification, set of classifications, or probability distribution to instances
Unknown instance is routed down the tree
Nominal:
number of children usually equal to number values
⇒ attribute won’t get tested more than once
Other possibility: division into two subsets

Numeric:
test whether value is greater or less than constant
⇒ attribute may get tested several times
Other possibility: three-way split (or multi-way split)
  Integer: less than, equal to, greater than
  Real: below, within, above
Does absence of value have some significance?
Yes ⇒ “missing” is a separate value
No ⇒ “missing” must be treated in a special way
   Solution A: assign instance to most popular branch
   Solution B: split instance into pieces
      Pieces receive weight according to fraction of training instances that go down each branch
      Classifications from leave nodes are combined using the weights that have percolated to them
Classification rules

Popular alternative to decision trees

*Antecedent* (pre-condition): a series of tests (just like the tests at the nodes of a decision tree)

Tests are usually logically ANDed together (but may also be general logical expressions)

*Consequent* (conclusion): classes, set of classes, or probability distribution assigned by rule

Individual rules are often logically ORed together

Conflicts arise if different conclusions apply
Easy: converting a tree into a set of rules

One rule for each leaf:
- Antecedent contains a condition for every node on the path from the root to the leaf
- Consequent is class assigned by the leaf

Produces rules that are unambiguous
- Doesn’t matter in which order they are executed

But: resulting rules are unnecessarily complex
- Pruning to remove redundant tests/rules
More difficult: transforming a rule set into a tree

Tree cannot easily express disjunction between rules

Example: rules which test different attributes

If a and b then x
If c and d then x

Symmetry needs to be broken

Corresponding tree contains identical subtrees

(⇒ “replicated subtree problem”)
A tree for a simple disjunction
The exclusive-or problem

If \( x = 1 \) and \( y = 0 \) then class = a
If \( x = 0 \) and \( y = 1 \) then class = a
If \( x = 0 \) and \( y = 0 \) then class = b
If \( x = 1 \) and \( y = 1 \) then class = b
If $x = 1$ and $y = 1$
then class = $a$
If $z = 1$ and $w = 1$
then class = $a$
Otherwise class = $b$
Are rules independent pieces of knowledge? (It seems easy to add a rule to an existing rule base.)

Problem: ignores how rules are executed

Two ways of executing a rule set:

Ordered set of rules ("decision list")
- Order is important for interpretation

Unordered set of rules
- Rules may overlap and lead to different conclusions for the same instance
Interpreting rules

What if two or more rules conflict?
  Give no conclusion at all?
  Go with rule that is most popular on training data?
  …

What if no rule applies to a test instance?
  Give no conclusion at all?
  Go with class that is most frequent in training data?
  …
Special case: boolean class

Assumption: if instance does not belong to class “yes”, it belongs to class “no”

Trick: only learn rules for class “yes” and use default rule for “no”

Order of rules is not important. No conflicts!

Rule can be written in *disjunctive normal form*.

If \( x = 1 \) and \( y = 1 \) then class = a
If \( z = 1 \) and \( w = 1 \) then class = a
Otherwise class = b
Association rules

... can predict any attribute and combinations of attributes

... are not intended to be used together as a set

Problem: immense number of possible associations

Output needs to be restricted to show only the most predictive associations ⇒ only those with high support and high confidence
Support and confidence of a rule

Support: number of instances predicted correctly
Confidence: number of correct predictions, as proportion of all instances that rule applies to

Example: 4 cool days with normal humidity

If temperature = cool then humidity = normal

Support = 4, confidence = 100%

Normally: minimum support and confidence pre-specified (e.g. 58 rules with support ≥ 2 and confidence ≥ 95% for weather data)
Interpreting association rules

Interpretation is not obvious:

If windy = false and play = no then outlook = sunny
and humidity = high

is not the same as

If windy = false and play = no then outlook = sunny
If windy = false and play = no then humidity = high

It means that the following also holds:

If humidity = high and windy = false and play = no
then outlook = sunny
Rules with exceptions

Idea: allow rules to have *exceptions*

Example: rule for iris data

If petal-length $\geq 2.45$ and petal-length $< 4.45$ then Iris-versicolor

New instance:

<table>
<thead>
<tr>
<th>Sepal length</th>
<th>Sepal width</th>
<th>Petal length</th>
<th>Petal width</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>3.5</td>
<td>2.6</td>
<td>0.2</td>
<td>Iris-setosa</td>
</tr>
</tbody>
</table>

Modified rule:

If petal-length $\geq 2.45$ and petal-length $< 4.45$ then Iris-versicolor

**EXCEPT** if petal-width $< 1.0$ then Iris-setosa
default: Iris-setosa
except if petal-length ≥ 2.45 and petal-length < 5.355
    and petal-width < 1.75
    then Iris-versicolor
        except if petal-length ≥ 4.95 and petal-width < 1.55
            then Iris-virginica
        else if sepal-length < 4.95 and sepal-width ≥ 2.45
            then Iris-virginica
    else if petal-length ≥ 3.35
        then Iris-virginica
            except if petal-length < 4.85 and sepal-length < 5.95
                then Iris-versicolor
Advantages of using exceptions

Rules can be updated incrementally
  Easy to incorporate new data
  Easy to incorporate domain knowledge
People often think in terms of exceptions
Each conclusion can be considered just in the context of rules and exceptions that lead to it
  Locality property is important for understanding large rule sets
  “Normal” rule sets don’t offer this advantage
Default...except if...then...
is logically equivalent to
if...then...else
(where the else specifies what the default did)

But: exceptions offer a psychological advantage
Assumption: defaults and tests early on apply
more widely than exceptions further down
Exceptions reflect special cases
Rules involving relations

So far: all rules involved comparing an attribute-value to a constant (e.g. temperature < 45)

These rules are called “propositional” because they have the same expressive power as propositional logic

What if problem involves relationships between examples (e.g. family tree problem from above)?

Can’t be expressed with propositional rules

More expressive representation required
The shapes problem

Target concept: *standing up*

Shaded: *standing*

Unshaded: *lying*
A propositional solution

<table>
<thead>
<tr>
<th>Width</th>
<th>Height</th>
<th>Sides</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>Standing</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>Standing</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>Lying</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>3</td>
<td>Standing</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>3</td>
<td>Lying</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
<td>Standing</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>4</td>
<td>Lying</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>3</td>
<td>Lying</td>
</tr>
</tbody>
</table>

If width ≥ 3.5 and height < 7.0 then lying
If height ≥ 3.5 then standing
A relational solution

Comparing attributes with each other

If width > height then lying
If height > width then standing

Generalizes better to new data

Standard relations: =, <, >

But: learning relational rules is costly

Simple solution: add extra attributes
(e.g. a binary attribute is width < height?)
Rules with variables

Using variables and multiple relations:

If $\text{height_and_width_of}(x,h,w)$ and $h > w$
then $\text{standing}(x)$

The top of a tower of blocks is standing:

If $\text{height_and_width_of}(x,h,w)$ and $h > w$
and $\text{is_top_of}(y,x)$
then $\text{standing}(x)$

The whole tower is standing:

If $\text{is_top_of}(x,z)$ and
$\text{height_and_width_of}(z,h,w)$ and $h > w$
and $\text{is_rest_of}(x,y)$ and $\text{standing}(y)$
then $\text{standing}(x)$
If empty$(x)$ then $\text{standing}(x)$

Recursive definition!
Recursive definition can be seen as logic program
Techniques for learning logic programs stem from the area of “inductive logic programming” (ILP)
But: recursive definitions are hard to learn
   Also: few practical problems require recursion
   Thus: many ILP techniques are restricted to non-recursive definitions to make learning easier
Regression: the process of computing an expression that predicts a numeric quantity

Regression tree: “decision tree” where each leaf predicts a numeric quantity

- Predicted value is average value of training instances that reach the leaf

Model tree: “regression tree” with linear regression models at the leaf nodes

- Linear patches approximate continuous function
Linear regression for the CPU data

$$PRP = -56.1 + 0.049 \text{ MYCT} + 0.015 \text{ MMIN} + 0.006 \text{ MMAX} + 0.630 \text{ CACH} - 0.270 \text{ CHMIN} + 1.46 \text{ CHMAX}$$
Regression tree for the CPU data
Model tree for the CPU data

- **CHMIN**
  - <= 7.5
  - > 7.5

  - **CACH**
    - <= 8.5
    - > 8.5

    - **MMAX**
      - <= 28000
      - > 28000

    - **LM4 (50/22.1%)**
    - **LM5 (21/45.5%)**
    - **LM6 (23/63.5%)**

    - **LM1 (65/7.32%)**
    - **LM2 (26/6.37%)**
    - **LM3 (24/14.5%)**
Simplest form of learning: *rote learning*

Training instances are searched for instance that most closely resembles new instance

The instances themselves represent the knowledge

Also called *instance-based* learning

Similarity function defines what’s “learned”

Instance-based learning is *lazy* learning

Methods: *nearest-neighbor, k-nearest-neighbor*, …
The distance function

Simplest case: one numeric attribute
  Distance is the difference between the two attribute values involved (or a function thereof)

Several numeric attributes: normally, Euclidean distance is used and attributes are normalized

Nominal attributes: distance is set to 1 if values are different, 0 if they are equal

Are all attributes equally important?
  Weighting the attributes might be necessary
Learning prototypes

Only those instances involved in a decision need to be stored
Noisy instances should be filtered out
Idea: only use prototypical examples
Rectangular generalizations

Nearest-neighbor rule is used outside rectangles

Rectangles are rules! (But they can be more conservative than “normal” rules.)

Nested rectangles are rules with exceptions
Representing clusters I

**Simple 2-D representation**

**Venn diagram**

Overlapping clusters
**Probabilistic assignment**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.4</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>b</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>c</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>d</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>e</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>f</td>
<td>0.1</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>g</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>h</td>
<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

...  

**Dendrogram**

NB: dendron is the Greek word for tree